The General Radio Experimenter

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An Equal-Arm Capacitance Bridge

By Robert F. Field

Summary—The effect of a grounded shield and the Wagner ground connection upon the bridge are discussed, and the substitution method for measuring the capacitance and resistance of condensers is described.

The Type 216 Capacity Bridge is a shielded equal-arm bridge with self-contained shielded input and output transformers. Its circuit connections are shown in Figure 2. The bridge is balanced by a simultaneous adjustment of one of the condensers C_A or C_B, and the resistor R which may be placed in series with the condenser having the lower resistance by means of a two-point switch. For silence in the telephones the conditions of balance are

$$\frac{M}{N} = \frac{C_B}{C_A} = \frac{R_A}{R_B} \qquad (1)$$

and, since the ratio arms are nominally equal,

$$C_A = C_B \text{ and } R_A = R_B \qquad (2)$$

where the resistor R is included in either R_A or R_B. The setting of the resistor R indicates the difference in resistance between the two condensers; therefore both the capacitance and resistance of one must be known in order that the capacitance and resistance of the other may be calculated.

The different arms of the bridge, the condensers being compared, the power source, and the balance detector may all have capacitances to ground, and to each other. These effects render the simple bridge equations (1) incorrect because neither the currents in the two equal arms nor the currents in the two condensers need be equal to produce silence in the telephone receivers. There are two general methods of minimizing the effects of ground and mutual capacitances, namely the use of the Wagner ground and of shielding.

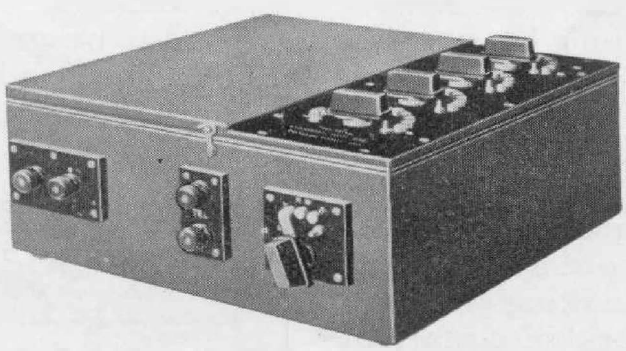

Figure 1. The Type 216 Capacity Bridge

The simplest type of Wagner ground consists (Figure 3) of a resistance PQ having an adjustable ground tap. By connecting a telephone receiver between the junctions of P and Q and of M and N, the latter junction may be brought to ground potential. Stratton* has discussed the use of the Wagner ground at considerable length and shows that the various ground capacitances, whether distributed or lumped,

* J. A. Stratton, "A High-Frequency Bridge," *Journal of the Optical Society*, XIII, October, 1926, 481.

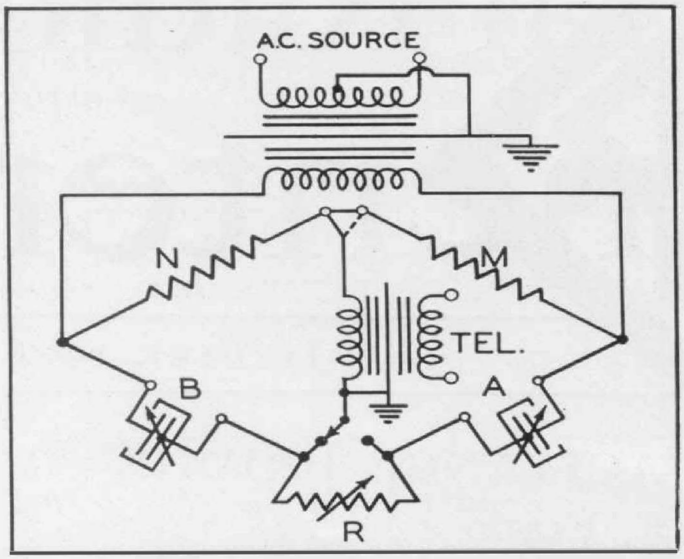

FIGURE 2. Wiring diagram for TYPE 216 Capacity Bridge

are equivalent to a resistance and capacitance, either in series or parallel, connected between each generator terminal and ground. These equivalent impedances may then be made to have the same ratio as the bridge arms *M* and *N* by a proper adjustment of the Wagner ground. For a complete adjustment, two variable condensers (shown dotted in Figure 3) must be connected in parallel with the ground resistance *PQ*. The two condensers may be made into one unit by rotating a single semicircular rotor between two insulated semicircular stators placed opposite each other. The values of the bridge arms are slightly altered if they have a distributed ground capacitance, but this is a second order effect. It may be eliminated by making the arms symmetrical, for the bridge equations are unaltered if *M* and *N* are equal impedances. Thus the Wagner ground minimizes the effect of fixed ground capacitances. It does not eliminate the effect of variable ground capacitances due to the change of position of the observer, the effect of mutual capacitances, and the effect of various voltages induced electrostatically or magnetically from outside fields. These effects may be reduced only by shielding.

In the TYPE 216 Capacity Bridge, this consists of a metallic grounded shield placed around the component parts of the bridge. The two equal arms need not be separately shielded if well spaced and kept symmetrical. The condensers being compared must be completely shielded and connected with their shielded terminals next to the detector. A condenser is of no value as a standard of capacitance unless this capacitance is independent of its position with respect to ground. Thus the use of the Wagner ground and the use of shielding are complementary and the one cannot easily supplant the other. The Wagner ground must be accompanied by moderate shielding of the bridge to eliminate induced voltages and body capacitance of the observer. On the other hand, the Wagner ground can be dispensed with if the bridge is shielded from the generating source and the balance detector by transformers whose primary and secondary windings are shielded by a grounded metallic sheath placed between them. If the bridge have equal ratio arms the center of the primary may also be grounded, and both primary and secondary made symmetrical with respect to the grounded iron core. If this shielding were perfect so that

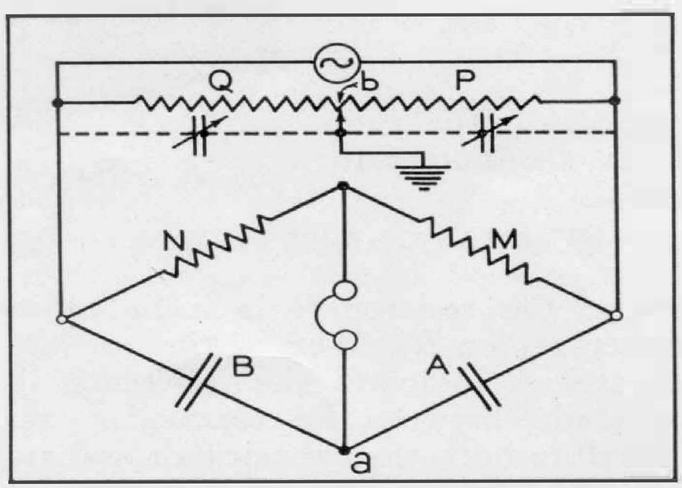

FIGURE 3. Wagner ground applied to an equal-arm bridge. When adjusting the Wagner ground one terminal of the telephone receivers is moved from point *a* to point *b*

there was no capacitance between primary and secondary, it would be necessary to shield only one transformer, but it is easier in practice to use less care and shield both. It is still essential, however, that the transformer and bridge be absolutely symmetrical, for otherwise the simple bridge equations cannot hold.

There are three possible ways of correctly comparing two condensers on a bridge thus shielded, even when the transformer and bridge are not symmetrical and the shielding of the input and output transformers is not perfect. First, a Wagner ground may be added across the secondary of the input transformer. Second, the condensers under comparison may be transposed. This method to a first approximation eliminates the value of the ratio arms M and N and gives as the value of the unknown capacitance the geometric mean of the two settings of the known capacitance. These values will be so nearly equal that their arithmetic mean may be used instead.

$$C_B = \frac{C_A + C_A'}{2} \text{ and } R_B = R_A + \frac{R + R'}{2} \cdot \quad (3)$$

Third, a substitution method may be used. In its simplest form the condensers under comparison are in turn connected in one arm of the bridge, while in the adjacent arm is placed a balancing condenser of equal capacitance which need not be calibrated. The bridge is balanced by adjusting the balancing condenser with the unknown in circuit, and by adjusting the known condenser with the known in circuit. This method is liable to error because of the use of two different means of balancing the bridge and because, if the condensers compared are dissimilar, their different capacitances to ground will affect the result unless both are perfectly shielded.

A modification of this method, which eliminates these difficulties and has at the same time an important advantage, consists in always keeping the known variable condenser in circuit. First the bridge is balanced with the two condensers, known and unknown, in parallel. Then the unknown condenser is disconnected on its unshielded side and the known condenser increased to produce balance again. The change in capacitance of the known condenser for the two positions of balance is the capacitance of the unknown condenser.

$$C_x = C_s' - C_s, \quad (4)$$

where C_s is the capacitance of the known or standard condenser for the first balance when the unknown condenser is in circuit and C_s' is its capacitance when the unknown condenser is out of circuit. The TYPE 222 Precision Condenser is suitable for use as the known or standard condenser. Any TYPE 246 Condenser or the TYPE 239–J Condenser may be used as the balancing condenser. With this equipment, condensers of less than 1500 mmf. capacitance may be measured to within

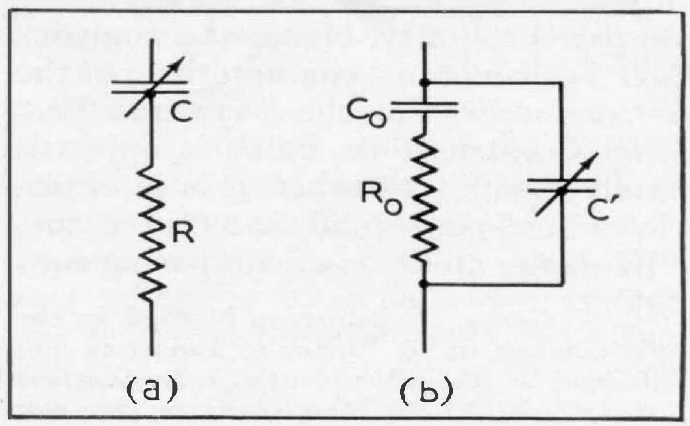

FIGURE 4

1 mmf., using the calibration chart provided with the TYPE 222 Precision Condenser.

Any air condenser is equivalent to two condensers in parallel: the one, a perfect variable air condenser having no energy losses; the other a fixed condenser (due to the solid dielectric) having all the energy losses. Since a condenser having an energy loss is equivalent to a perfect condenser in series with pure resistance, an air condenser having capacitance C and resistance R (Figure 4a) may be represented as in Figure 4b, where C_0 and R_0 are the capacitance and resistance of the solid dielectric condenser respectively and C' is the capacitance of the variable air condenser. Burke has shown * that

$$R\omega C^2 = R_0\omega C_0 \text{ and } C = C_0 + C'$$

provided that the power factor of C is negligible, i.e.:

$$R_0\omega C_0 << 1.$$

If the capacitance of the solid dielectric condenser does not change with the setting of the air condenser,† the quantity $R\omega C^2$ is constant at any fixed frequency because R_0 and C_0 are constant. For different frequencies the quantity $R_0\omega C_0$ which is the power factor of the solid dielectric, is approximately constant for good dielectrics, such as isolantite, porcelain, or hard rubber.‡ For such materials $R_0\omega C_0$ is of the order of 0.005, whose square is negligible compared to unity. Hence the quantity $R\omega C^2$ is a constant, characteristic of the air condenser. For the TYPE 222 Precision Condenser its value is approximately 0.06 x 10^{-12} when R is in ohms, ω in radians per second, and C in farads.

In all the methods of comparing condensers previously discussed, except the last, it was found that the resistance of the standard condenser must be known in order that the resistance of the unknown condenser might be determined. In the modified substitution method last described it is only necessary to know the law of variation of the resistance of the standard condenser with setting. This is due to the fact that this condenser is kept in circuit during both

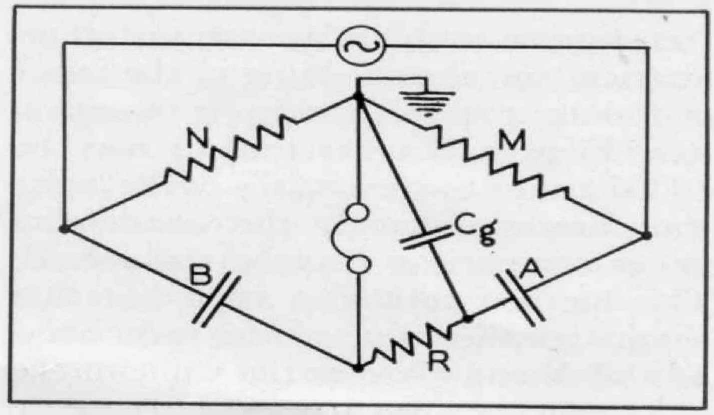

FIGURE 5

measurements. It may be shown * that the resistance of the unknown condenser is given by

$$R_x = (R' - R)\frac{C_s'^2}{C_x^2} \qquad (6)$$

where C_x is the capacitance of the unknown condenser obtained from equation (4); $C_s'^2$ is the total capacity in that arm of the bridge; and $R' - R$ is the change in resistance of the added resistor R of Figure 2, always taken as positive. If for these two settings the added resistor is transferred from one arm to the other, the sum of its reading rather than their difference must be taken.

There are two points at which a shielded bridge may be grounded to the shield, the junction of the ratio arms and the junction of the shielded sides of the two condensers. If it were not necessary to introduce a resistance between the two condensers to obtain the resistance balance, the junction

* C. T. Burke, "Substitution Method for the Determination of Resistance of Inductors and Capacitors at Radio Frequencies," *Transactions of A. I. E. E.*, XLVI, May, 1927, 483. See also Appendix I.

† This can be brought about by so building the air condenser that the electric field in the solid dielectric supports is independent of the condenser setting. The General Radio TYPE 222, TYPE 246, and TYPE 239 Condensers meet this requirement.

‡ An excellent discussion of the relation between the resistance and power factor of condensers is found in "Radio Instruments and Measurements," *Circular of the Bureau of Standards No. 74* (1st ed., 1918, or 2nd ed., 1924) pp. 122 to 129. (Government Printing Office, Washington, D. C.)

* Appendix II.

of the ratio arms would be the better point at which to place the ground, because then the capacitances to ground of the input transformer and bridge arms are placed in parallel with the ratio arms. But the ground capacitance of the condenser, in series with which the added resistance is placed, is a shunt across both the added resistance and the detector and turns the bridge into a triple network as shown in Figure 5. It may be shown* that while the effect on the capacitance is negligible, the true resistance is given by

$$R_x = (R' - R)\left(1 + \frac{C_g}{C}\right)\frac{C^2}{C_x^2}. \qquad (7)$$

Thus for a given ground capacitance C_g, the resistance R_x as calculated by the simple formula, equation (6), is always less than the true resistance and approaches it as the total capacitance C is increased. Its true value may be approximated by taking a set of observations for different values of total capacitance C and plotting the resistance as calculated from equation (6) against the reciprocal of the total capacitance. The intercept of this curve on the axis of resistance, where total capacitance is infinite, is the true resistance. The error introduced by this effect is greatest when the bridge and condensers are placed on a grounded table, and may easily amount to 50 per cent. for small values of the total capacitance.

When the ground is placed at the junction of the added resistance and the shielded side of one of the condensers, this correction is eliminated and the true resistance of the unknown condenser is given by equation (6). The ground capacitance of the condenser, in series with which the added resistance is placed, is a shunt across the added resistance only and has a negligible effect. But the capacitances to ground of the input transformer and bridge arms are now placed in parallel with the balancing and standard condensers, and any methods of comparing condensers involving a knowledge of the total capacitance of either condenser arm must be corrected for this added capacitance.

The value of capacitance which can be measured by the modified substitution method is limited to the range of the standard condenser. This is about 1400 mmf. for the TYPE 222 Precision Condenser. This limit may be raised in the following ways. Consider first the calibration of a variable air condenser of maximum capacitance greater than, but less than twice, this limit. It is calibrated in the ordinary manner up to the limit, using equations (4) and (6). Let the largest value of capacitance and the corresponding resistance be C_a and R_a respectively. Without changing the setting of the unknown condenser it is connected in circuit, the standard condenser is set at approximately maximum, the balancing condenser is increased correspondingly, and the bridge is balanced. Now of course the unknown condenser cannot be disconnected and the bridge balanced as usual by increasing the standard condenser, but the capacitance and resistance necessary for such a balance may be calculated from the previous measurement. Let C' and R' be this necessary capacitance and resistance respectively, and let C and R be the observed values of capacitance and added resistance after the capacitance was increased. Then, since for both cases the unknown capacitance and resistance are unchanged

$$C_a = C' - C \text{ and } R_a = (R' - R)\frac{C'^2}{C_a^2}$$

$$\text{or } C' = C_a + C \text{ and } R' = R_a\frac{C_a^2}{C'^2} + R. \qquad (8)$$

* Credit for the original derivation of this case should be given to Mr. R. P. Siskind of the Department of Electrical Engineering, Harvard University.

These new values C' and R' may now be used for the remainder of the calibration as the capacitance and added resistance respectively, which would balance the bridge if the unknown condenser were disconnected. For a still larger condenser this process may be repeated.

Two condensers each larger than the standard variable condenser may be compared by this method if the difference of their capacitances is less than the capacitance range of the standard condenser. Two measurements are made, one with each condenser connected in parallel with the standard condenser. Let C_1 and R_1, C_2 and R_2 be the capacitance and resistance respectively of the two condensers, and C_s and R, C_s' and R' the corresponding capacitance of the standard condenser and the value of the added resistance respectively. It may be shown* that

$$C_2 = C_1 + (C_s - C_s')$$

$$R_2 = R_1 \frac{C_1^2}{C_2^2} + (R - R') \frac{(C_s + C_1)^2}{C_2^2}. \quad (9)$$

The limitation that the difference of the capacitances of the two large condensers must be less than the capacitance range of the standard condenser may be removed by the use of an additional

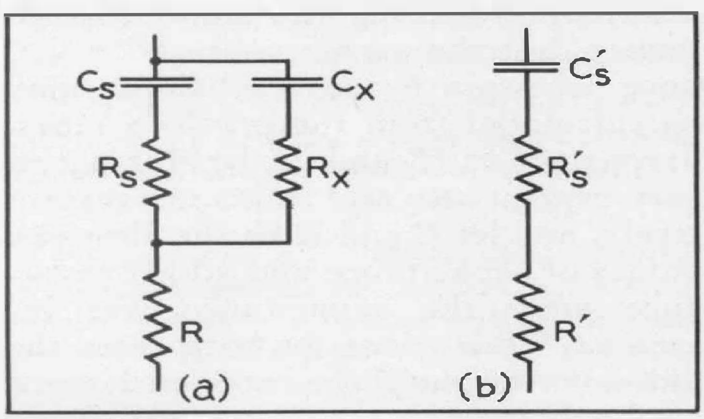

FIGURE 6

condenser of sufficient size to bridge this gap, whose capacitance and resistance are known. This method may be

* Appendix II.

applied to the calibration of a set of condensers arranged to form a decade or a number of decades in terms of one of their number or in terms of the standard variable condenser. The decades may consist of ten similar units, or of four or more separate units of different values, such as the combinations 1, 2, 2, 5; 1, 2, 3, 4; and 1, 2, 3, 6.

In order to obtain resistance measurements of reasonable accuracy it is necessary to observe certain precau-

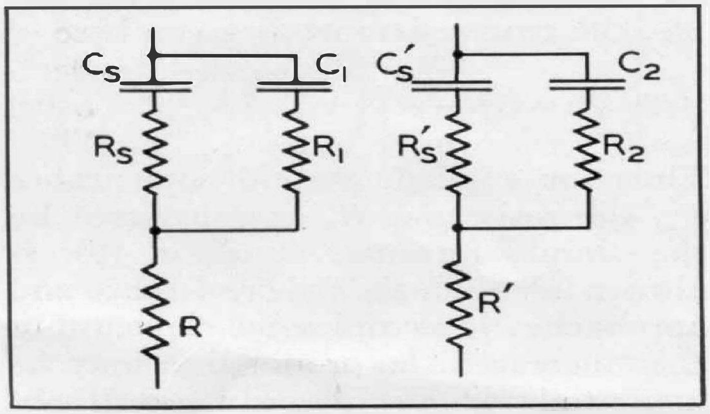

FIGURE 7

tions which are perhaps not as important for capacitance measurement. Due to the small power factor associated with the usual condenser, the capacitance balance must be made with great accuracy, much greater than that to which the standard condenser can be read or calibrated, except when very large capacitances are being compared. For the typical case of a total capacitance of 1000 mmf. the capacitance balance must be adjusted to .01 mmf. in order that the resistance balance may be made to 1 ohm. To attain this sensitivity a potential of at least 50 volts at a frequency of 1000 cycles must be applied to the bridge and a two-stage amplifier connected to the telephone transformer. The wire to the unknown condenser must remain in place and connected to the unshielded terminal of the standard condenser when the unknown is disconnected, its motion must be kept small, and, in order to keep its capacitance and dielectric losses small, it should be uninsulated.

Since the resistance of a condenser varies inversely as the frequency, the frequency of the voltage applied to the bridge must be constant within much closer limits than are required for resistance or inductance measurements. The fluctuations of frequency must not be great enough to change the resistance balance by 1 ohm. This effect will be the greater, the larger the losses in the unknown condenser, for then the compensating effect of the resistance of the balancing condenser will be less. The generating source should also be as free from harmonics as possible. When the bridge is balanced for the fundamental, the resistance balance for the second harmonic and for all other harmonics is quite incorrect. Hence any harmonics in the source will be heard in the telephones in proportion to their magnitude. Due to their different pitch the trained ear can discriminate against them, but it is always easier to effect a bridge balance when they are small.

Occasionally, especially when measuring condensers containing poor dielectrics, the bridge balances, both capacitance and resistance, will appear to drift with time. This may be due to a frequency shift, but it is more likely due to an increasing temperature of the dielectric produced by its own energy losses. The temperature coefficient of resistance of solid dielectrics is large and for accurate measurements some sort of temperature control is necessary.

APPENDIX I

The general law for n condensers in parallel may be easily derived from a consideration of the energy relations involved. The power loss in a condenser is

$$W = I^2 R.$$

For the ordinary negligibility relation, namely $R^2 << X^2$,

$$I = \frac{E}{X} = E\omega C,$$

whence the loss in watts per squared volt is

$$\frac{W}{E^2} = R\omega^2 C^2.$$

Since power is additive,

$$\begin{aligned}\frac{W}{E^2} &= R\omega^2 C^2 \\ &= R_1\omega^2 C_1^2 + R_2\omega^2 C_2^2 + \cdots + R_n\omega^2 C^2_n \\ &= \sum_1^n R_m\omega^2 C_m^2,\end{aligned}$$

or

$$R\omega C^2 = \sum_1^n R_m\omega C_m^2 \qquad (10)$$

and

$$R = \frac{\sum_1^n R_m\omega C_m^2}{\omega \sum_1^n C_m^2}. \qquad (11)$$

APPENDIX II

In the modified substitution method of comparing two condensers the two arrangements shown in Figure 6 must have identical impedances when the bridge is balanced for both. In (a) the unknown condenser is in circuit, in (b) it is disconnected. Constants of the standard condenser are indicated by subscript s and the added resistance is R. Primed letters indicate the second balance, when the unknown is disconnected. For identical impedances,

$$C_s + C_x = C_s'$$

$$R + \frac{R_s\omega C_s^2 + R_x\omega C_x^2}{\omega (C_s + C_x)^2} = R' + R_s'. \qquad (12)$$

Now, from the law of the air condenser (Equation 5)

$$R_s\omega C_s^2 = R_s'\omega C_s'^2$$

$$C_x = C_s' - C_s \text{ and } R_x = (R' - R)\frac{C_s'^2}{C_x^2}. \qquad (13)$$

In the comparison of two large condensers by the modified substitution method, the two arrangements shown in Figure 7 must have identical impedances.

$$C_s + C_1 = C_s' + C_2$$

$$R + \frac{R_s\omega C_s^2 + R_1\omega C_1^2}{\omega\,(C_s + C_1)^2} = R' + \frac{R_s'\omega C_s' + R_2\omega C_2^2}{\omega\,(C_s' + C_2)^2}. \quad (14)$$

From the law of the air condenser (Equation 5)

$$R_s\omega C_s^2 = R_s'\omega C_s'^2$$

$$C_2 = C_1 + (C_s - C_s')$$

$$R_2 = R_1\frac{C_1^2}{C_2^2} + (R - R')\frac{(C_s + C_1)^2}{C_2^2}. \quad (15)$$

MISCELLANY

By THE EDITOR

IN spite of our promise to include in this issue a description of General Radio quartz plates, we are deferring it until the February issue so that we may publish in full Mr. Field's discussion of capacitance and condenser loss measurements. We feel that the subject is of general enough interest to justify this change in our plans.

* * * *

The TYPE 216 Capacity Bridge described by Mr. Field in this issue is now being manufactured with the ground connection made to the junction of the shielded sides of the two condensers. Those who have bridges in which the connection is made to the junction of the ratio arms may make the change themselves or have it done by the General Radio Company for $5.00.

* * * *

The price of the TYPE 481 Polar Relay described in the *Experimenter* for February, 1929 has been reduced from $30.00 to $25.00. The code word is NOMAD.

CONTRIBUTORS

A brief biographical sketch of ROBERT F. FIELD, the author of "An Equal-Arm Capacitance Bridge," appeared in the Miscellany column of the *Experimenter* for last October.

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